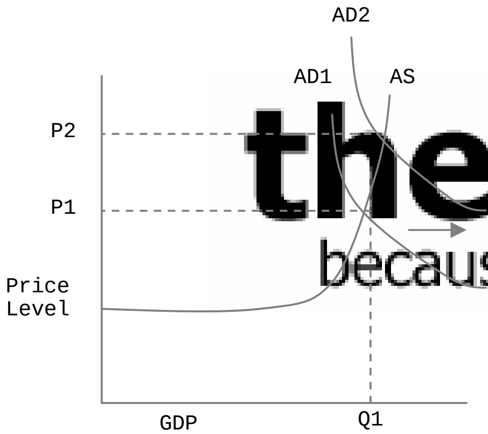
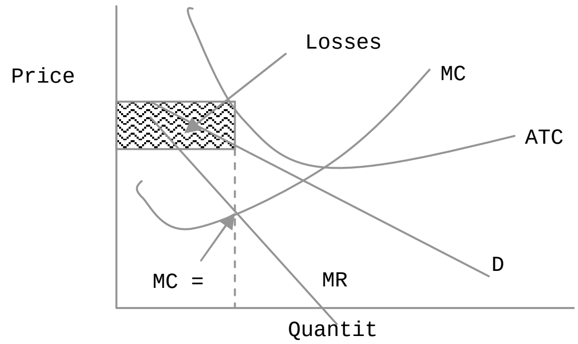
the short run

because in the long run, everybody's dead





Finance Review Guide

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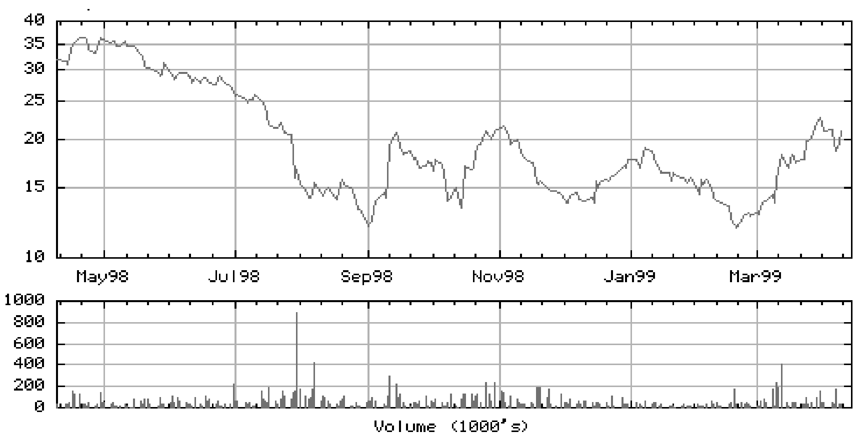

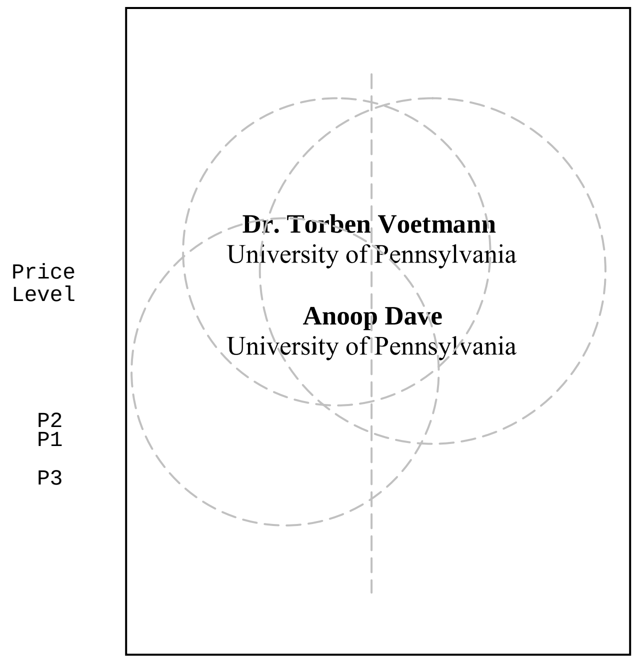

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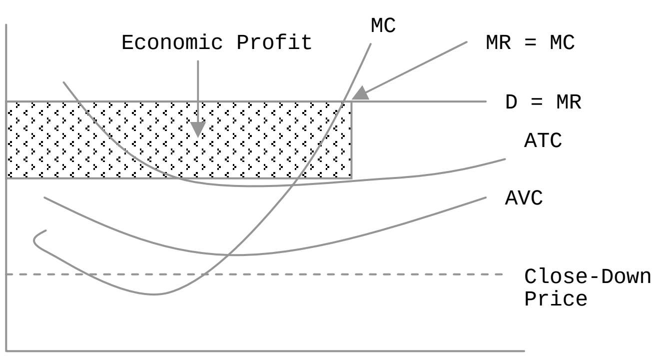
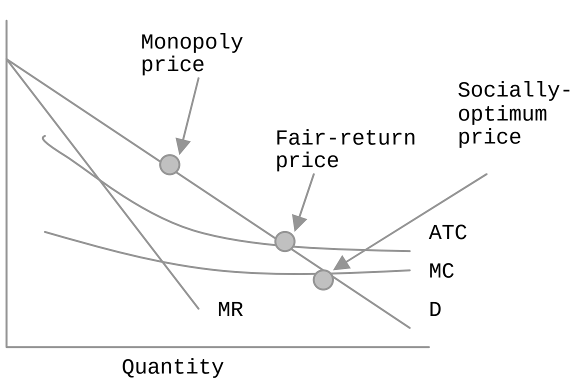




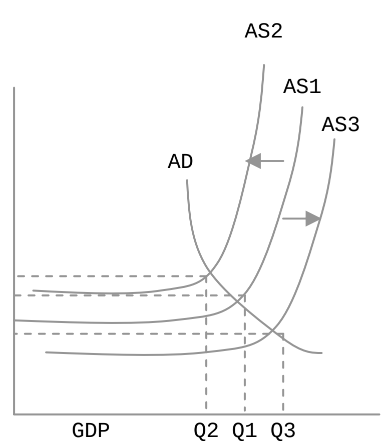


Table of Contents

- Basic Tools...4
- Cash Flows...6
- Risk Preferences...10
- Discounting...13
- CAPM/SML/CML...19
- Capital Structure ...24
- Efficient Market Hypothesis/Corporate Valuation ...28
- Options...33

Basic Tools

Purpose: Throughout this guide various mathematical concepts will be used. This portion of the guide serves to teach and refresh the most basic concepts.

Average (Arithmetic Mean): A statistic calculated by summing a set of data values and dividing by the number of values

Example: What is the average of: 3, 45, -5, 64, and .25?

Solution: 3+45+-5+64+.25 = 107.5
N (the number of elements in the set) equals 5. 107.25/5 = 21.45
The average of the set is 21.45

Expected Value (weighted average): A statistic calculated by summing up a set of data values and multiplying them by their respective weights or probabilities:

Example: A company will have profits of \$300 during a boom, \$200 during normal conditions, and -\$500 during a recession. The probabilities of boom, normal period, and recession are .2, .5, and .3 respectively. What is the expected profit of this company?

Solution: (300)(.2)+(200)(.5)+(-500)(.3) = \$10

Variance: A statistic which measures how spread out or dispersed a set of data is. The value calculated will always be greater than or equal to zero, with larger values corresponding to data which is more spread out. If all data values are identical, the variance is equal to zero. Variance is often used as a proxy for risk.

Example 1: Calculate the variance of the data set in the average example:

Mean= 21.45

$$Variance = \frac{(3 - 21.45)\hat{}2 + (45 - 21.45)\hat{}2 + (-5 - 21.45)\hat{}2 + (64 - 21.45)\hat{}2 + (.25 - 21.45)\hat{}2}{5}$$

Variance= 770.91

Example 2: Calculate the variance of the data set in the expected value example:

Mean = 10

$Variance = (.2)(300\text{-}10)^2 + (.5)(200\text{-}10)^2 + (.3)(\text{-}500)^2$

Variance = 112900

Standard Deviation: The square root of the variance

Adding Standard Deviations: To add two standard deviations first square the standard deviations so you have the variances. Next add the two variances and then add two times the covariance. Covariance equals the correlation coefficient times the two standard deviations.

Forumula:

ρ = Covariance(a,b)/($\sigma_a\sigma_b$)

ρ = correlation coefficient (between 1 and –1). This number describes how a set of data move together

σ = standard deviation

Example: Adding two variances

The standard deviation of data set A is 4 and the standard deviation of data set B is 5. The covariance of the two data sets is 2. If a portfolio contains both data sets A and B (and the data sets are weighted equally in the portfolio), what is the variance of the combined portfolio?

Solution:
Variance A equals: 16
Variance B equals: 25
2*Cov(a,b) = 2*2 = 4

$(.5^2)16+(.5^2)25+4(.5)(.5) = 11.25$

We perform the calculations $.5^2 * 16$ and $.5^2 * 25$ because both A and B compose ½ of the portfolio. We square these because the standard deviations compose ½ of the portfolio, but to add standard deviations we have to square the standard deviations (calculate variance) and then add the variances together plus the covariance term. The covariance term is also multiplied by the respective weights of A and B.

Variance of the portfolio equals 11.25 and the standard deviation of the portfolio is 3.35

General Formula: Adding Variances

K_a^2*Variance (A) + K_b^2*Variance (B) + 2 * K_a * K_b * Covariance (A, B)

K = weight of A or B

Cash Flows[1]

Purpose: Finance is the trade off of two things: risk (fear) and return (opportunity). Cash flows characterize the opportunity side of this equation.

Free Cash Flows (FCF): Total cash available for distribution to owners and creditors after funding all pertinent investment activities.

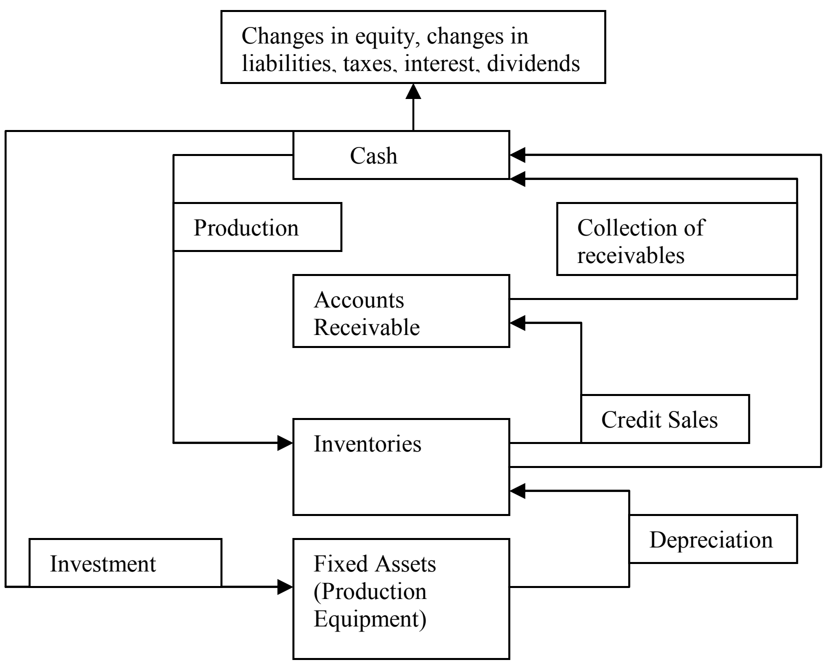
Why are free cash flows used?

Everything a company has is cash in a certain form. Inventories for goods that will be sold will become cash upon sale. Manufacturing equipment will aid in producing goods that will later be transformed into cash. Some of these things are somewhat illiquid, the most liquid of all assets is cash. If for some reason a company cannot produce enough liquid cash to cover expenses, then insolvency can occur –a firmwill not be able to pay its cash obligations. This will result in insolvency and the firm will have to eventually shut down operations.

Caution: Just because a firm is profitable does not mean a firm will produce enough cash flow to cover its obligations. If a firm is highly profitable because it has profits from sales on accounts receivable[2], it may not have enough cash to cover expenses.

Model: The Cash Cycle:

This model illustrates the movement of cash in a firm[3]:



[1] For more depth information about free cash flows read Chapters 1-3 of Corporate Finance: A Valuation Approach by Benninga and Sarig.
[2] Accounts receivable (A/R) are promised payments to the firm. That is, the firm is promised by a buyer that s/he will pay for the goods given to him or her. Cash is not collected at the time of the promised sale.
[3] Adopted from Analysis for Financial Management 6th Edition by Robert C. Higgins

Summary of the Cash Cycle:

Cash is used to buy raw materials, labor, etc. These factors are then used to produce goods that are stored in the inventory. Inventory is depleted through direct sales (which generate cash) or sold on credit (accounts receivable). Eventually payment is made for the accounts receivable sales, and cash is generated. This portion of the model is the working capital cycle.

The other activity of the cash flow cycle is the investment cycle. Cash is used to buy production equipment (fixed assets). These assets depreciate over time from general wear and tear. It is as if a portion of these fixed assets go into every unit they create. For this reason, we have put an arrow going from fixed assets to the inventory.

Cash is also used for other expenses. That is for taxes, interest payments, dividend payments, etc. We will dicuss changes in equity and liabilities in later sections.

Calculating Free Cash Flows (FCF)

Free cash flows are calcuated using information from the financial statements. Note, FCF must be derived from manipulation of the financial statements. The financial statements are created on an *accural basis* meaning they recognize revenue *not* when payment is made but when an intent to pay has been made. Three major financial statements exist:

<u>Balance Sheet:</u> This statement is a listing of assets and the financing of these assets. It follows the equation Assets = Liabilities + Shareholder's Equity. In short, total assets must equal total liabilities and shareholder's (also termed as owner's equity) equity.

Model: Breakdown of the balance sheet[4]

Assets	Liabilities and Shareholder's Equity
Current Assets: Short term assets. *Cash*- money the firm has in the bank *Marketable Securities*- securities held in place of cash by the firm *Accounts Receivable*- unpaid bills to the firm *Inventories*- parts in hand, parts in storage, parts in the process of being finished (works in progress), that are not yet sold	**Current Liabilities:** Short term liabilities. *Accounts Payable* (A/P)- unpaid bills to suppliers *Accrued Taxes* –unpaid taxes *Current Portion of Long Term Debt*- Portion of long-term debt that must be paid off in a year *Short-Term Borrowing*- all principle of debt that, in principle, has to be repaid in a year
Fixed Assets: *Leased Property, Plant, and Equipment (PPE)*- if the firm has long term leases the property may appear on the sheet as if it were owned by the firm. *PPE*: Listed at the cost of acquisition minus depreciation: This is PPE actually owned, not leased, by the firm. *Land*	**Long Term Liabilities:** *Obligations under leases*- Corresponds to leased property, plant, and equipment. Long term financial leases. *Long-Term Debt*- borrowing done by the firm to be repaid after many years. **Preferred Stock**
Goodwill: If assets that have been acquired for more than market value, the excess is counted as goodwill. This is typical when a firm buys another company – the acquiring firm pays a price over the value of the acquired.	**Equity:** Investment in firm by owners. *Stock Value:* amount owners paid for original stock value of a firm *Retained Earnings:* profits after taxes that are not paid as dividends.
Sum of this is TOTAL ASSETS	Sum of this is TOTAL LIABILITIES

[4] Adopted from Corporate Finance: A Valuation Approach by Benninga and Sarig.

Income Statement: While the balance sheet is a snapshot in time, that is the balance sheet states all the firm owns and owes on one exact date, the income statement records cash flow over time.

Note 1: The operatating segment reports the result of the companies major ongoing activies while the non-operatating segment summarizes all secondary activities.

Note 2: Earnings are often also called profits, income, net profit, or net income. Net sales are often called revenues or net revenues.

Income Statement

Sales: Periodic (annual, monthly, etc.) sales of the company

Cost of Good Sold: (COGS) cost of making the goods

Selling, General, and Administrative: (also known as SGA) Other operating expenses

Interest: Periodic cost of money cost from loans (or bonds)

Depreciation: This is often calculated from a schedule from the IRS. Other methods to calculate depreciation exist. This is a non-cash expense. We will delve deeper into this when calculating FCF

Taxes: Allowance for taxes to be paid in the period. Sometimes taxes are recorded as current expenses, but will be paid at a later period. These are recorded as deferred taxes and appear on the balance sheet as a long-term liability

Dividends: Cash paid to shareholders of a firm. This is often separated as dividends to preferred holders (who get dividends first) and common shareholders (who can get paid after preferred receive a base amount)

Retained Earnings: These earnings are added to accumulated earnings of the firm. Added to Retained Earnings in the balance sheet

Income Statement

Net Sales:	800	
-Cost of Sales		400
Gross Profit	400	
-Selling Expenses		200
-General and Administration Expenses		50
-Depreciation and Amortization		20
-Amortization of Goodwill		2
Total operating expenses		272
Operating Income	128	
-Interest Expense		10
-Other Expense		18
Total non-operating expenses		28
Income before Income Tax	100	
Provision for Tax	35	
Net Income	65	

Cash Flow Statement: This is derived from the income statement and changes in the balance sheet. This statement has 3 parts: cash flow from operations, cash flow from investment activities, and cash flow from financing activities. Our primary interest are FCF and calculating them are somewhat different than the cash flow in the cash flow statement.

Calculating Free Cash Flows:

There are two main methods for calculating FCF: the direct method and the indirect method. Both should result in the same answer. Usually the information at hand will dictate which method is used.

FCF (direct)		
	Sales	Sales also include credit sales so this must be corrected
	-Increase in A/R	to do this we subtract increases in A/R
-Operating Expenses	-COGS	
	-SGA	
	-Increases in inventories	Inventory is paid for with cash
	+Increase in A/P	Expenses are not yet paid
	+Depreciation	This is not a cash expense and is taken out
-Cash Operating Taxes	-Tax on income	
	+Increase in Taxes Payable	Increases in taxes payable saves the company cash
=Cash from Operations	Sum of the above	because they do not have to pay this year
-Net Increase in PP&E	-Inc in PPE	
	=Free Cash Flow	

Note on depreciation: The depreciation expense schedule is usually designed by the IRS. The purpose of this expense is to stablize cash flows from year to year. For an example if the company buys a $100 million piece of equipment in year one, its profits for year one will appear very low. In the following years, the profit will seem to rocket, but that is only because the difference in year one and year two was the $100 million cost. The IRS balances out this one-time cost over many years to steady the streams of profits for different years. The job of the IRS is to collect taxes; they do not care about Free Cash Flows, rather they care about collecting a fair share of taxes.

FCF: The indirect method

Profits After Taxes	
-Increases in Accounts Receivable	
-Increase in Inventories	
+Increase in Accounts Payable	
+Increase in Taxes Payable	
+After-Tax Interest Expense	This is done so we can evaluate the operations side and the financial side of a company separately. Interest payments are done on debt. We are calculating OPERATING cash flows
=Cash from Operations	
-Increases in PPE	
=Free Cash Flow	

Risk Preferences

Now that we have introduced the opportunity side of finance, let us continue with the risk side. Below is an example to demonstrate how risk can change the value of cash, depending on the type of person looking at the situation.

Example: Imagine two companies. Company A will make $100 during good times and $0 during bad times. Company B, which is more robust, will make $50 during both good times and bad times. Which company would you choose?

Risk neutral, Risk seeking, Risk averse (adverse), and the Utility Function

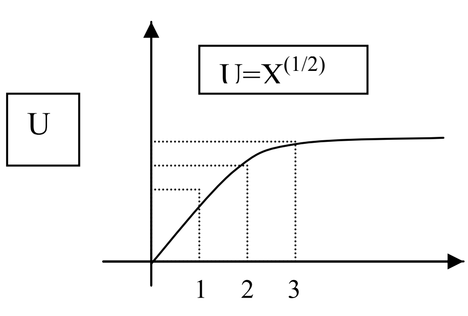
If we look at expected values of A and B, we see that they are both equal: .5*100+.5*0 = 50 and .5*50+.50*50 = 50. Basically, for B you will get a gauranteed cash flow of 50, and for A you may either get nothing or you may get 100. Yet certain people tend to prefer A over B and another group of people prefer B over A. A third type of people are indifferent to either A or B. Those who prefer A are **risk seeking,** those who prefer B are **risk averse**, and those that are indifferent to A and B are **risk neutral**. In essence, the previous question had no "right" or "wrong" answer.

People have different preferences when it comes to risk, but the question is can we quantify that perception of risk?

It is impossible to sum up a single person's likes and dislikes into one simple formula. Statisticians and economists have created general equations to characterize the preferences of these three types of people. The premise of these equations is that dollars generate satisfaction or value. This utility is measured by the utility function.

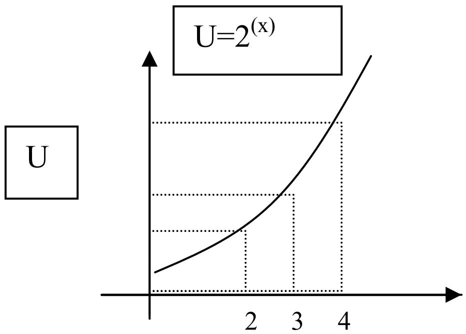
Utility Function for Risk Averse Person

The utility function of a risk averse function would be something to the effect of $U=X^{(1/2)}$. X would be an amount in dollars and U is the utility generated.



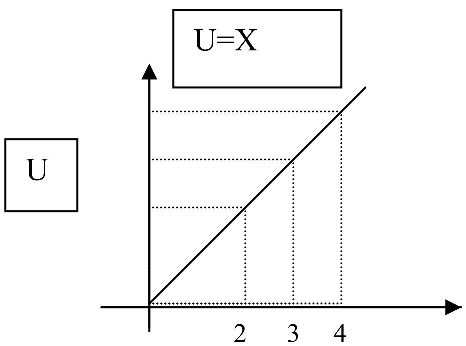
The graph helps to illustrate the characteristics of a risk averse person. Notice that as X increases so does U. That is, a risk averse person has increased utility at point X=2 than compared to point X=1 (U has a higher value at X=2 than X=1). But, U increases by a lesser increment as X increases. That is the U jump from X=1 to X=2 is more than the jump from X=2 to X=3. To verify this, note that $2^{(1/2)}-1^{(1/2)}$ is greater than $3^{(1/2)}-2^{(1/2)}$. This leads us to conclude that an absolute higher value of X will generate more utility for a risk averse person, but a risk averse person will is more concerned about losing a dollar than gaining an extra dollar. The latter conclusion comes from the fact the increments of U's increase by decreasing amounts. Each dollar gain gives less marginal utility than the prior dollar.

Utility Function for Risk Seeking Person



This graph illustrates the characteristics of a risk seeking individual. Just like the risk averse person as X increases, total U increases. That is total utility is greater at X=3 than X=2. Basically, the more money a risk averse or risk seeking person has the happier he becomes. Contrary to the risk averse person, a risk seeking person's happiness increases by increasing amounts. This is to say the U increase from X=2 to X=3 is less than the increase from X=3 to X=4. To verify this plug these numbers into the utility function. You will get 8-4<16-8. A risk seeking person gets more marginal utility from each dollar gained rather than each dollar lost. While the risk averse person cares more about losing a dollar, the risk seeking person cares more about gaining an extra dollar.

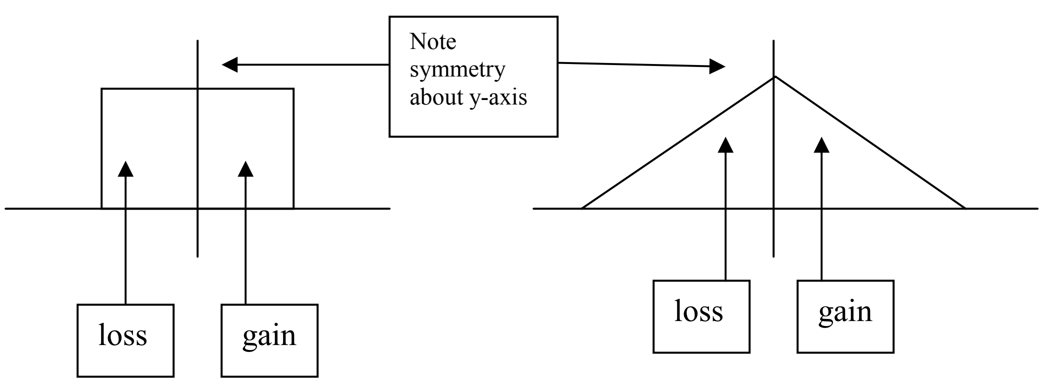
Utility Function for Risk Neutral Person



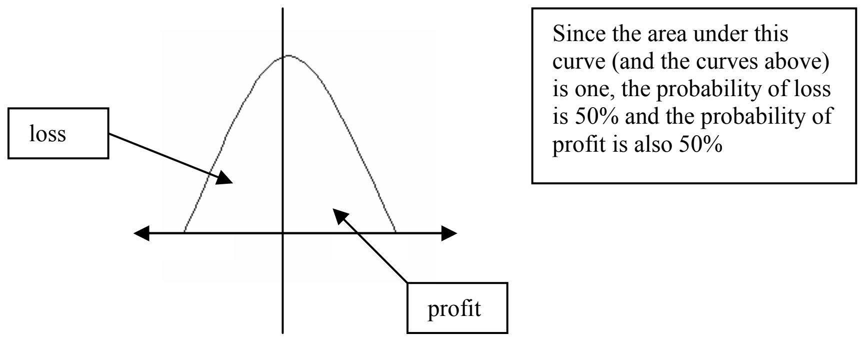
This graph illustrates the characteristics of a risk neutral person. Like the previous to graphs, notice that as X increases the total value of U increases. That is X=2 yields a smaller U than X=3. The change in marginal utility per increase in X, however, is constant. That is, the jump from X=2 to X=3 increased U by an increment of 1. The jump of U from X=4 to X=3 also increased U by an increment of 1. The risk averse person is indifferent to gaining an extra dollar or losing a dollar. Most of finance assumes risk neutrality or risk aversion. In this guide we will assume risk neutrality, unless otherwise stated.

Variance as a proxy for risk

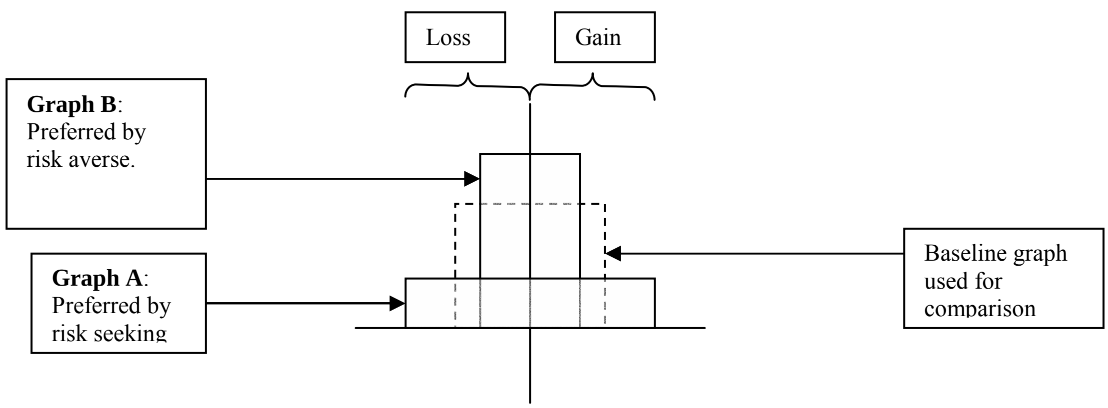
Oftentimes, people will use variance (or standard deviation, which is the square root or variance) as a proxy for risk. That is they will equate increased variance with increased risk. In most circumstances this is a reasonable measure. The conditions that make this proxy a valid assumption are those when we have a symmetric distribution. This means that for a certain gain, there must be an equal complimentary loss. If you have a 50% chance of losing at least $20 you must have a 50% chance of gaining at least $20. Below some symmetric pay off graphs.



Note: These graphs are symmetric distributions but not Gaussian Normal Distribution graphs. The normal curve (also called the Bell Curve) is a symmetric graph with mean equal to zero and a standard deviation equal to one. While a normal curve is a symmetric graph not all symmetric graphs are normal curves. Also we will assume the area under the curve of all these graphs is 1. Below we have a drawing of a normal curve.



Because of symmetry we can use variance as a proxy for risk, we can also see what type of payoff graph various investors would like.



Above are graphs with the same area (1) and same expected values (i.e. the means are all zero). The differences in the graphs are that the variances are different. Graph A has a smaller variance, that means there is less upside potential, but there is also less downside potential. Graph B has more upside potential, but also carries more downside potential. A risk neutral person would be indifferent to Graph A, B, or the baseline graph, because a risk neutral person does not take into account risk.

When is variance not a good proxy for risk?

Asymmetric graphs cannot have variance as their proxy for risk because they have different properties. Options (which will be discussed later) are an example of an asymmetric payoff structure. With a put or call option the minimum value of the option is zero –the option expires. The maximum value of a call option is theoretically infinite. As variance increases the chance of attaining a higher payoff also increases, but there is no downside risk (the minimum value of an option is zero). For these situations, variance is not a good proxy for risk.

Discounting

Discounting is the heart of finance. It incorporates the time value of money and the opportunity cost of money.

Compounding/Future Value

One important concept in finance is compounding. It refers to the idea that you earn interest on money you have earned interest on before. If you had a $100 in the bank and earn 5% a year in interest, at the end of year one you have $105 (100*1.05). At the end of year two you get 5% on $105, which is 110.25 ($105*1.05$ or alternatively $100 * 1.05^2$). Year 3 you would have $115.76 ($110.25*1.05$ or alternatively $100 * 1.05^3$).

The general formula for compounding 'n' dollars at an 'r' interest rate for 'T' periods is:

Dollars after t periods of compounding = $n * (1+ r)^T$

Another name for this value is future value, for this equation we have replaced 'n' with C_t (this stands for cash flow at time t. T+t stands for cash flow in the future, specifically T periods from now).

$$\mathbf{FV = C_{t+T} = C_t * (1+ r)^T}$$

Below is an illustration of the concept of future value. Future value is equal to what present cash flows will equal at some time T given an interest rate r. The hashed arrows indicate that we are taking cash flow today and finding its value at a later time.

Time Period	t	t+1	t+T
General Form:	C_t	$C_t*(1+r)$	$C_t*(1+r)^2$
Numerical Example	1	1.05	1.10

Present Value/Discounting

The 'reverse' of future value is present value. This is the value at what money in the future is worth today. If we were to get $110.25 at the end of two years or $100 today (assuming that the rate we could invest the money was 5%) we would be indifferent between the two proposals. This is because we could take the $100 and invest it at 5% for 2 years and get the $110.25. Reversing the order of the terms for the FV equation we can get the Present Value equation:

$$\mathbf{PV = C_t = C_{t+T} / (1+ r)^T}$$

Below is an illustration of the concept of present value. Present value is equal to what future cash flows equal today (or at sometime prior to receiving the cash flows). The hashed arrows indicate that we are taking cash flow at some future point and finding its value at an earlier time. When we take the present value of a cash flow, we are **discounting** that cash flow.

Time Period	t		t+1		T
General Form:	$C_T/(1+r)^2$	←	$C_T/(1+r)$	←	C_T
Numerical Example	1	←	1.05	←	1.10

Example:
Imagine you can invest at 10% risk free. You can get \$250 in 5 years –what is the equivalent amount of money today?

Solution:
$250/(1.1)^5$
$= 155.23$

You would be indifferent between \$155.23 and \$250 in 5 years.

Example 2:

Imagine you get the following payments. \$5 in year 1, \$10 in year 2, and \$15 in year 3. What is the PV of this cash flow? The appropriate rate is 10%.

$PV = 5/1.1 + 10/(1.1^2) + 15/(1.1^3)$
$=24.08$

Caveat: The term period

One common mistake people make while discounting is forgeting that T and t stand for time periods, not years. Period can indicate month, day, year, century, etc. When doing finance problems be sure to pay special attention to when the compounding takes place. Oftentimes, you will be quoted an interest rate, but that interest rate will be compounded at some interval. The quoted interest rate is called the stated annual interest rate or the annual percentage rate, APR. Banks and other financial institutions use the APR term quite frequently.

Example: You can be told that you are earning 10% interest a year, but the cash is compounded quarterly (that is every three months). How much is \$100 worth at the end of the year?

Solution: Since compounding is occurring quarterly, we are traveling through four periods in one year –not just one.

First, we find the quarterly interest rate. Since 10% is not the rate we earn because of compounding, we can take 10% and divide it by 4.

Next, we will list all the information we have:

$10\%/4 = 2.5\%$
$r= 2.5\%$

t=4
CF_t = $100

Plug in the numbers in the FV equation:

$100 * $(1.025)^4$ = $110.38

Note that this is slightly more than earning 10% over one year: that would have given us a final value of $110 (100*1.1). We earned 38 cents more because of the compounding.

Formula: A general statement is that compounding an investment x times a year provides an end of year value

$$\mathbf{CF_T = CF_t (1 + r/x)^x}$$

Whenever using this formula, pay special attention to the compounding period!

Caveat 2: Calculating the Interest rate on compounded dollars

Notice in the example above we divided the 10% APR rate by four then went about the calculation. This was because the 10% was not a compounded rate. Had it been compounded we would have had to take the 4^{th} root of one plus the rate because the rate had been compounded. The example below clarifies this concept.

Example: Suppose you have earned 25% on your investment over four years. Your investment is compounded annually. What is the interest rate per year for your investment?

Solution: You have earned 25% over four years, and this 25% is your total return. That means if you invested $100, at the end of four years you would have $125 dollars. Since your investment compounds annually, you went through 4 compounding periods.

Mathematically we can display this as: $125 = 100(1+x)^4$
We solve for x.

$125/100 = (1+x)^4$
$1.25^{1/4}=(1+x)$
$1.0573=(1+x)$
$x= .0573$

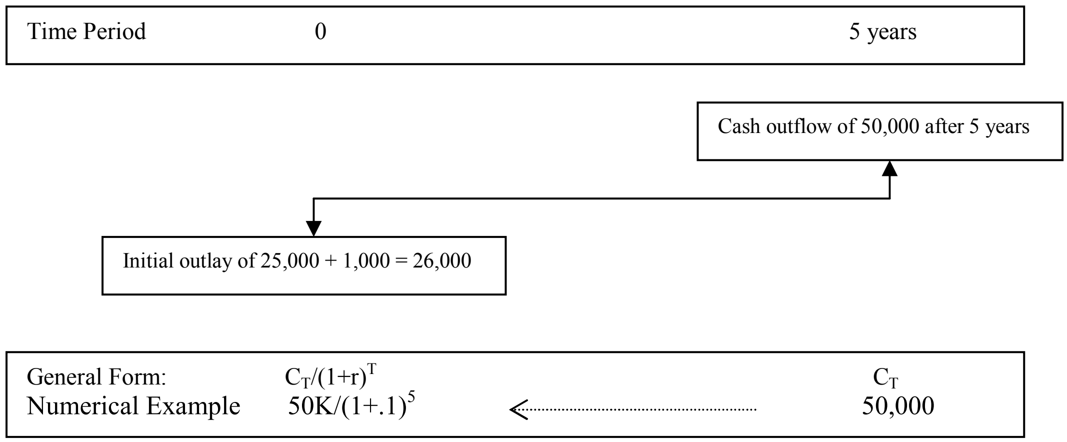
You have been earning 5.73% on your investment per year.

Net Present Value (NPV)

The basic idea of NPV is that you should only undertake investment decisions that prove to be more profitable what the market offers. If you can get a better return from alternative sources (the financial markets for an instance), you are better off putting your money in them instead of the proposed investment project.

Example: A tele-marketer calls you up and informs you that he has a great plan that will double your $25,000 in savings in 5 years. All you need to do is pay the firm $1,000 for “sound investment advice”. Assuming the marketer’s plan works, should you take him up on the deal, if markets are offering you a 10% return? Assume the risk of the tele-marketer’s project is the same as that of investing in the market.

Solution: The way to approach the problem is to find the net present value. Net present value means we will take the present value of the opportunity and subtract it from the initial outflow. Below is a graphical depiction.



Since the risk of the project is the same as that of investing in the market, we can use 10% as the appropriate discount rate. Always discount cash flows by their appropriate risk factor! Many students will discount by 10% because they feel the alternative to investing in the project is investing in the stock market. This is not correct. If the project was less risky than investing in the markets, then it should be discounted by a rate less than the market.

$PV = 50{,}000/(1.1)^5$
$= 31046.07$
We want to compare this to the initial cost of the project.
Total cost = 25,000 + 1,000 = 26,000
NPV = 31046.07-26,000 = 5046.07

The NPV is positive because the PV of the 50,000 is worth having 31046.07 today. We are essentially paying 26,000 for 31046.07. This is a good project.

An alternative way to compare the projects is to see what would happen in five years. We could take 26,000 and invest it directly in the market.

$26{,}000(1.1^5) = 41873.75$

This is 8126.74 less than getting 50,000 from the tele-marketer's deal. The astute reader will note that 8126.74 is the future value of 5046.07 ($5046.07*1.1^5$). Realize that this alternative calculation gives us FV not NPV.

IRR

A common term in finance is IRR –internal rate of return. The IRR is the rate at which NPV equals zero. You can calculate the IRR of a project through a computer program or by using algebra.

Example: Calculate the IRR of a project that costs 100 and pays 50 in year 1, 50 in year 2, and 50 in year 3.

Solution: Using a calculator program or by doing simple algebra solve the following equation
$0 = -100 + 50/X + 50/X^2 + 50/X^3$
X= 1.2337
The IRR is 23.37%

The rules for IRR are (assuming K is the percentage cost of capital):

IRR > K : accept the investment
IRR < K : reject the investment
IRR = K : the investment has a zero NPV, it is a marginal investment

Usually IRR and NPV will yield the same results. If the cash flows are irregular (such as you pay at the end of the investment period, but not at the beginning), then IRR will yield a result that is different from NPV. NPV is a more precise valuation method, but since IRR is used frequently, it is a good technique to know, but be weary of its limitations.

Rule of 72

A good quick tool to use when r is relatively small is the rule of 72. This rule of thumb is a quick way to figure out how many periods it will take for your money to double if invested at a given interest rate r. The formula is r*period =72. If you are earning 6% (compounded annually), then you will double your investment in about 12 years. Remember, this is a simple trick and not a precise method!

Annuity

An annuity is the payment of a fixed amount for a certain number of periods. Getting $50 for the next five years is an example of an annuity. One way to get the present value of such a payment would be to take the present value of each cash flow. This could be tedious. A simple way to do this is to use the annuity formula.

Formula: PV= C{1/r – 1/[r(1+r)]}

C is the constant stream of cash flows. If you are getting $50 a year C = 50.

If you are getting a constant stream of cash flows that grows at a constant rate, (for example assume your cash flows are growing at the rate of inflation) then you should use the growing annuity formula. An example of constant growing cash flows are: 50, 52.5, 55.125, 57.88. These cash flows are increasing by 5% a year.

Formula: $PV= C\{1/(r-g) – (1+g)^n/[(r-g)(1+r)^n]\}$

g is the growth rate. In the 50, 52.5, 55.125, 57.88 stream, g= .05.

Perpetuity

A perpetuity is a constant stream of payments received per period forever. Some lottery games promise to pay you a certain amount of money, say $5000 for the rest of your life. These lottery games are offering you a perpetual cash stream. To value this type of perpetuity the following equation is useful:

Formula: PV = CF/r

Growing perpetuities are cash flows that grow (for example with inflation) every year. If you get a 2% growing perpetuity and your first payment is $5000, then you would get 5000, 5100, 5202, 5306.04, etc.

Formula: PV= CF/(r-g)

The formula of a perpetuity and growing perpetuity is derived from the fact discounting (with r<1) is similar to a converging geometric series. You can derive this formula with a bit of calculus. This is just a fun fact, and not necessary for the scope of this text.

Problems in the "Real World"

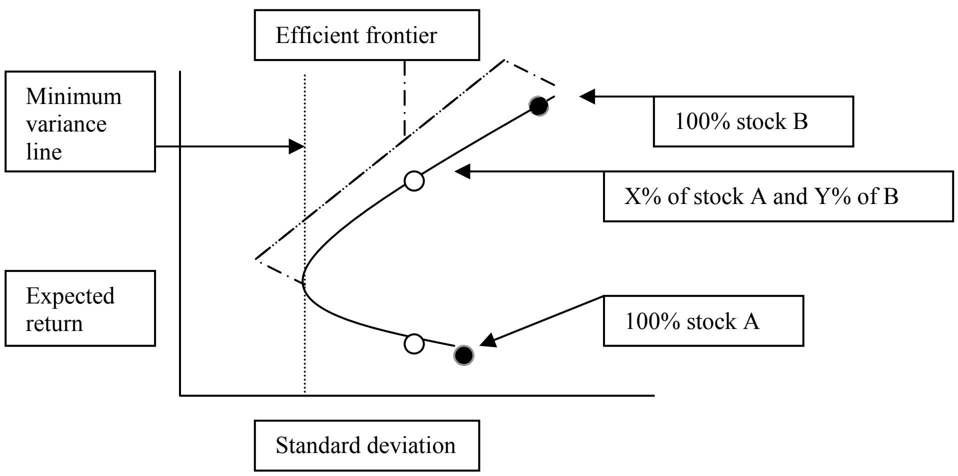
We have seen that we should always accept positive NPV projects. In many corporations there is a separation between owners and managers. Owners (e.g. shareholders) sometimes do not do the daily tasks of managing. For this reason negative NPV projects are sometimes adopted due to some dubious reasons. We will explore this problem later in the text.

CAPM/SML/CML

This section is going to go in depth into the concept of risk for reward.

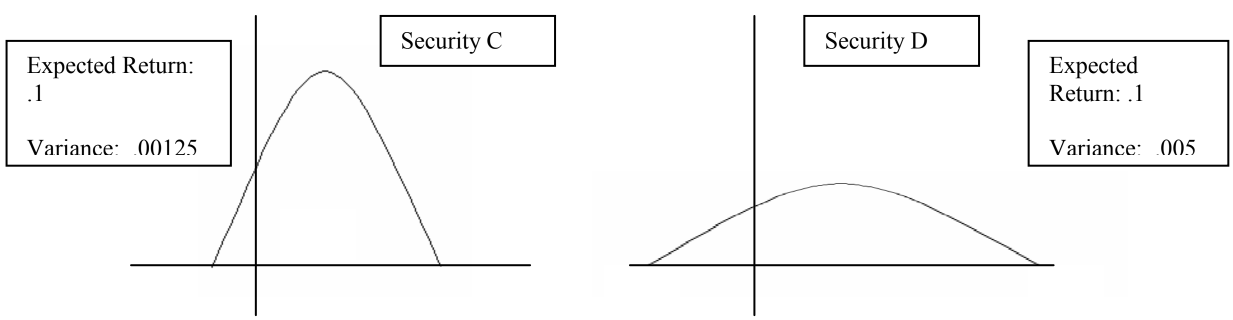
Efficient Frontier

Below is a graph of the efficient frontier. This graph has standard deviation on the x-axis and expected return on the y-axis. This graph shows the benefits of diversification. The various points on the line represent different combinations of securities in a portfolio. The minimum variance line is tangent to the left most point of the portfolio combination. It is at this point that the investor is exposed to the least amount of risk (standard deviation being the proxy for risk). Note the efficient frontier is the portion of the curve above the minimum variance point. That is because points below this minimum variance point are inefficient. Take a look at the two unshaded points on the graph, they both have the same standard deviation (call this standard deviation x), yet the unshaded point on the efficient frontier has a higher expected return. A person willing to take risk x would take the point on the efficient frontier as it provides a higher expected return for the same risk exposure.



Deriving The Frontier

The efficient frontier shows that various combinations of securities can yield different risk and return ratios. The reason for this phenomenon has to do with the pay off schemes of the different securities and the correlation between the two securities. Below are two pay off diagrams for two different securities.



Security C has the same expected value as security D, but security C has a smaller variance (the curve is less spread out) than security D.

If we invested 2/3 in C and 1/3 in D the expected return of the portfolio would be .1

The variance of the portfolio would depend on the covariance of the two securities. When adding the variance of two securities we must take into account the covariance of the two securities (see basic tools section for more information about adding variances). Recall that covariance is a scaled version of correlation (see basic tools section for formula). The correlation coefficient is between 1 (when C goes up D always goes up) and –1 (when C goes down D always goes down).

If the covariance between C and D is -.0025 then the variance of holding 1/3 D and 2/3 C is:

$(1/3)^2(.005)+(2/3)^2(.00125)+(2)(1/3)(2/3)(-.0025) = 0.$

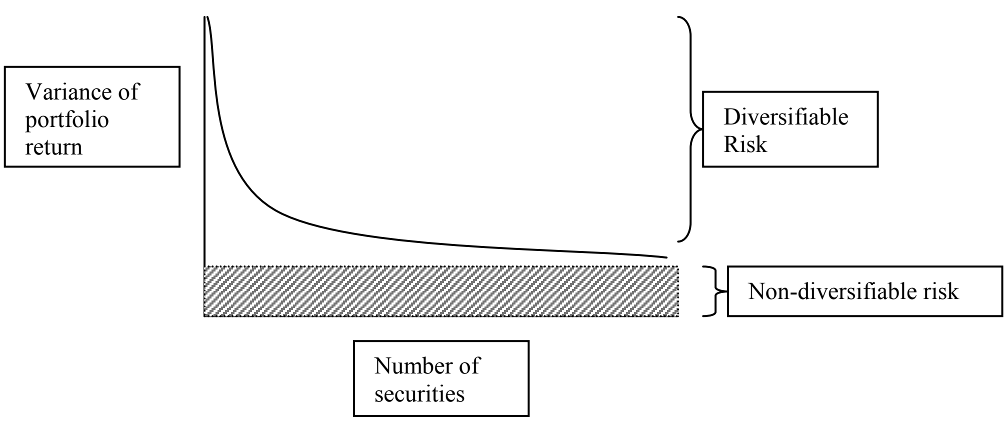
The above result shows that holding both securities exposes one to less risk than holding one security alone. The expected return is not diminished.

The efficient frontier is curve that has various combinations of such securities (such as 45% of one security 55% of another). **Note:** It is possible to do the above calculations with many more securities; we have only chosen two securities for didactic purposes.

Conclusions:

1) The expected return on a portfolio is the weighted average of the expected returns.
2) Contingent on the correlation coefficient the standard deviation of the portfolio could be less than the weighted average of two individual securities.

Systematic Vs. Unsystematic Risk



Assumptions for graph:
1) All securities have constant variance
2) All securities have constant covariance
3) All securities are equally weighted in portfolio

Significance

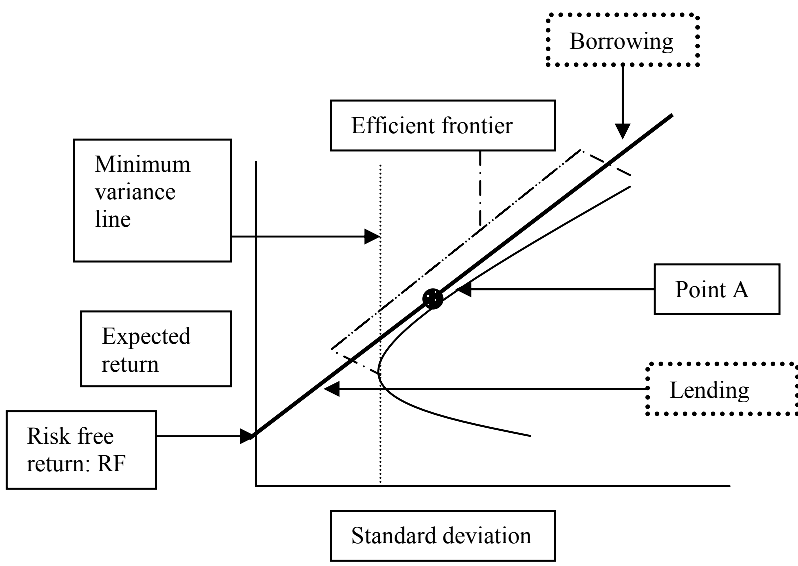
This graph shows the benefits of diversification. As the number of securities in a portfolio increase, the risk of the portfolio decreases (we saw this with the efficient frontier). The risk we can decrease as a result of buying many different types of securities is called "diversifiable risk" (also known as unique risk or unsystematic risk). For example, a recession may hurt a travel company because less people will have the money to travel, but firms that specialize in bankruptcy may get added business during a recession. Owning stock in both companies will diversify some of the risks associated with a recession (or a boom). Note that diversification can never be zero, there will always be some risk no matter how well diversified a portfolio is. This risk is called non-diversifiable risk (also called portfolio risk, market risk, systematic risk). It is basically the risk from operating in the "system". For example, all US firms will face the same political risks (i.e. Republicans or Democrats in the Senate).

Total risk of a security = Portfolio (systematic) risk + unsystematic (diversifiable) risk

Capital Market Line (CML)

The capital market line introduces the borrowing and lending dimension of our analysis.

Assumptions for CML
1) Riskless borrowing and lending. A risk free asset has a 0 standard deviation and therefore 0 covariance with any asset. Note risk free assets can still generate a return, US T-bills are considered risk free assets, yet they do have a return.
2) Homogenous expectations: this means all investors will identify the same efficient set and same CML and hold the same portfolio of risky assets. In the graph below this is described as point A.



The CML (the bolded line) is given by the equation $R_p = P_A R_A + (1-P_A) R_F$. P refers to a percentage invested in risky asset A. R_a is the return on asset A and R_f is the risk free return. CML states that an investor can combine riskfree assets and a risky asset causing him to be somewhere on the line. Point A is where the investor is neither lending nor borrowing, that is he is invested in only risky assets. This point is where the CML is tangent to the efficient frontier. Note that the CML provides a higher return for any given standard deviation (it provides an equivalent return at point A).

Seperation Principal and Borrowing and Lending:
This principal states that an investor's investment decision has two parts:

1) Calculating the efficient frontier and then finding tangent point A by determining the CML. The capital market line will always have y intercept Rf and have one point tangent to the efficient frontier. This portion of the analysis requires no assumptions on the risk profile (is the investor risk averse). It is the result of the calculations of variance and expected returns.

2) After point A is determined the investor's risk profile comes into play. If the investor is risk averse (wants a smaller standard deviation), he will start **lending** money (buying bonds) at the risk free rate. This will decrease his standard deviation exposure and expected return. He will lie at some point between the y intercept and point A of the CML. If he is risk seeking, then he will **borrow** more money and invest the amount in portfolio A. He will increase his expected return, but he will increase his risk exposure as well. The investor that borrows will be content at some point beyond A on the CML. The investor's placement on CML will be contigent on his risk profile (risk adverse or risk seeking).

Security Market Line and CAPM

Beta (β)

Formula:
$\beta = \sigma_{i,m}/\sigma^2_m$

The market portfolio is defined as the sum of all the securities in the market and the investors that hold them. A proxy for the market is the S&P 500. The relevant risk of any individual security is its contribution to the market portfolio. This is described by β. $\sigma_{i,m}$ is the covariance of the market and the security while σ^2_m is the variance of the market. The beta of the market portfolio is one, this makes intuitive sense because the weighted sum of all the securities is by definition the market portfolio. Beta is another proxy for risk for a single security in a large portfolio.

Variance vs. Beta

When will an investor view beta as the proper measure of risk and when will an investor view variance as the measure of risk?

Regardless of whether an investor holds one security or a diverse portfolio, variance (standard deviation squared) is the proper measure of risk. In a diversified portfolio the investor does not care about the variance of each security, rather he is interested in the contribution of the security to the variance of the portfolio. Under the homogenous expectations assumption all individuals hold the market portfolio. We measure the added risk by how much it increases the variance of the portfolio. If we standardize this contribution of variance we get the beta of the security.

Beta is the marginal contribution of risk to the portfolio.

CAPM –Capital Asset Pricing Model

The return we hope to earn from investing in the market is, $R_m \rightarrow R_m = R_f$ + Risk premium. R_m is the expected return on the market. This can differ from the actual return! Recall expected return is the weighted average return. R_f is the risk free rate.

A way to approximate the return (R) for a given security is given by: $R = R_f$ + risk premium * β. We can solve for risk premium from the above equation ($R_m = R_f$ + Risk premium) so that risk premium = R_m – Rf. We multiply the risk premium by beta because depending on how correlated the security is with the market, that is how responsive the security is to the market (how does the security move with the market, if the market goes up does this security also go up?). The final equation is the Capital Asset Pricing Model:

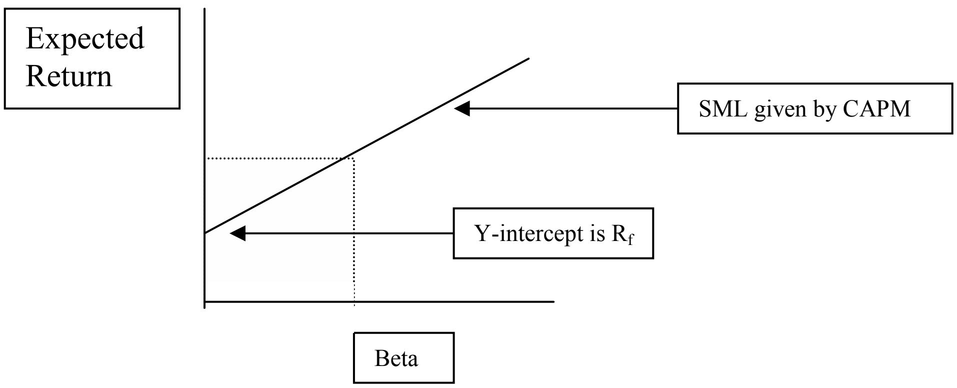
Formula
$\mathbf{R = R_f + \beta * (R_m - R_f)}$

R_m = expected market return
R_f = risk free rate

$(R_m – R_f)$ = risk premium, which is defined as the risk premium
$\beta = \sigma_{i,m}/\sigma^2_m$

Security Market Line: SML

When we graph CAPM the resulting line is called the Security Market Line.



Note: Where the hashed lines intersect on the SML is where Beta of the security = 1 and the expected return is equal to the expected return on the market.

CAPM is useful for determing the cost of equity (which we will go into more detail in the next section).

Ending marks: Difference between SML and CML

Students often confuse SML with CML. Below are the respective definitions:

SML: A line that displays the equilibrium relationship between systematic risk and expected return on individual securities. It represents the relationship between the expected return and the market risk (which is proxied by beta).

CML: The efficient set of all assets (risky and riskless) that gives an investor the best opportunities.

CML applies to a well diversified portfolio (hence risk is shown by standard deviation). SML applies to individual securities in a portfolio (hence risk is defined by beta).

Capital Structure

In this section we continue on our risk-return journey by applying the concepts taught in the previous section. We will figure out how discount rates are determined and use the information to calculate NPV and value a firm.

Formula (refresher):
NPV= (-Initial Investment) + $\sum[C_t/(1+r)^t]$

C_t = Cash flow at time t
r = discount rate (also called cost of capital)

CAPM –AGAIN

Recall from the Cash Flows section a firm has different uses for cash:

1) Invest cash in a project
2) Pay a dividend

A firm should only invest in projects that have positive NPVs, because these projects will add value to the firm. Sometimes, when a firm has a lot extra cash it succumbs to the temptation of empire building. It will start making frivolous expenses (extra corporate jets, more than necessary country club memberships, etc.). These expenses will not earn a return to satisfy the investment demands of the equity (stock) holders. The return the equity holders require is called the cost of equity. It would have been more prudent for the firm to pay out the extra cash in the form of dividends, rather than to squander it on foolish investments.

Calculating cost of equity (R_e):

We can calculate the cost of equity using the CAPM equation:

General CAPM Formula
$\mathbf{R = R_f + \beta * (R_m - R_f)}$

When calculating the R_e we must replace β with β_e (beta equity). This beta captures the risk associated with holding an equity position in the firm.

What Determines Beta?
Recall that beta is the measure of the systematic risk of a company. Below are the determinants.

(Systematic) Business Risk:

1) **Cyclicality**: Firms whose profits are strongly tied to the business cycle have high betas because they move more with the market.
2) **Operating Leverage**: The greater a firms commitment to fixed costs (costs that are not contingent on how much a firm produces), the greater the beta.

   This comes from the following rationale:
   PV(asset)= PV(revenue) – PV(fixed costs) – PV(variable costs)
   PV(revenue)= PV(fixed costs) + PV(variable costs) + PV(asset)

   Those who receive fixed costs tend to be debt holders. Debt holders are paid before equity holders hence equity holders have a decreased chance of receiving payment (they are in a secondary payment position).

(Systematic) Financial Risk:

1) **Financial leverage**: Increased financial leverage increases the beta because of the argument above –debtholders are paid before equity holders.

From this we get two important formulas:

Formula 1:

$\beta_{Asset} = D/V * \beta_{Debt} + E/V\ \beta_{Equity}$

D= Debt
E=Equity
V= D+E
Beta asset is the beta of the firm with no debt

Formula 2:

$\beta_{Equity} = \beta_{Asset}\,(1 + D/E * (1\text{-}Tc))$

*The above formula assumes risk free corporate debt. If the debt is not risk free use:

$\beta_{Equity} = \beta_{Asset} + (1\text{-}Tc)(\beta_{Asset}\ \beta_{Debt})(D/E)$

Tc = Corporate tax rate (expressed as a percentage)
D= Debt
E=Equity

Note: The equity beta will always be greater than the asset beta if there is leverage (the firm holds debt). The minimum value of beta equity is beta asset.

Tax Benefits of Debt:

Formula 2 above shows that taxes decrease beta equity. The higher the tax rate the lower the beta equity will be. The reason for this is that debt has a "tax shield". The example below shows how the quirk in the IRS tax code causes an increase in debt to be an increase in value.

Assume:

EBIT = 1000
Debt interest rate (R_d)=.1
D=Debt = 100
Tc = .5

	Firm Without Debt	Firm With Debt
EBIT	1000	1000
Interest (R_d*D)	0	-10
EBT (Earning before Taxes)	1000	990
Taxes	-500	-495
Earnings after taxes	500	495
Total cash to stock holders and debt holders	500	495+10=505

As this simple example shows the owners of the firm (the owners being equity holders and debt holders) get more money with increased leverage. This is because interest escapes corporate taxation.

Caveat: You cannot increase leverage indefinitely. As leverage goes up so does the **cost of financial distress**. This is the risk that the firm will not make enough money to pay its debt obligations. As the debt/equity ratio increases, so does the likelihood of financial distress. That means as D/E increases beyond a certain point (the point is different for every firm, but basically it is the point at which the tax shield does not add enough value to compensate for the cost of financial distress) the value of the tax shield cannot justify the increased cost of financial distress. The costs of financial distress are reputation, lawyer fees, bankruptcy etc.

R_{wacc} / Wacc

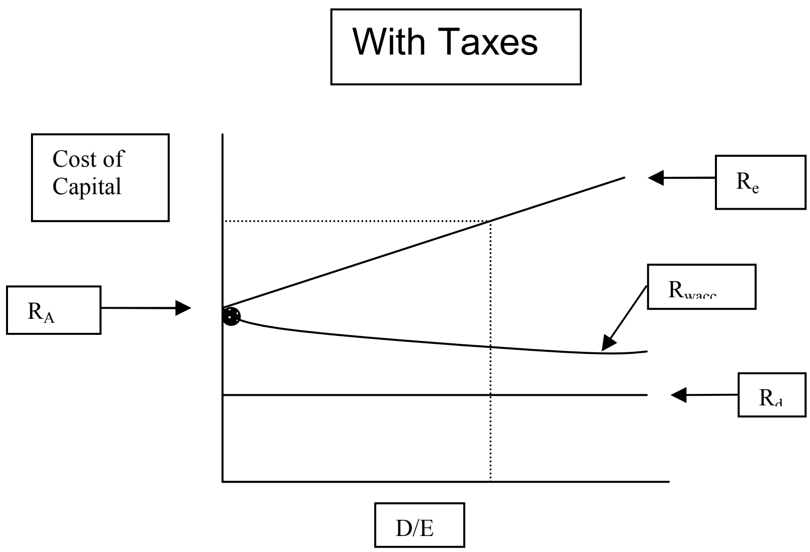
Since a firm has two types of stakeholders (debt holders and equity holders) it has two types of costs it is affected by. Those who lend debt are in a less risky position (they get paid first) and hence demand a lower return for their investment in the firm. (Lending money to a firm is similar to investing money in the firm for our purposes. This makes sense, because in both instances you are giving money to a firm and the firm then pays you back the amount you invested and a little bit more –hopefully. The difference is that equity has voting rights and other privileges.) Equity holders are in a riskier position (paid second) and hence demand a higher return (the other things that effect the beta equity have been covered previously).

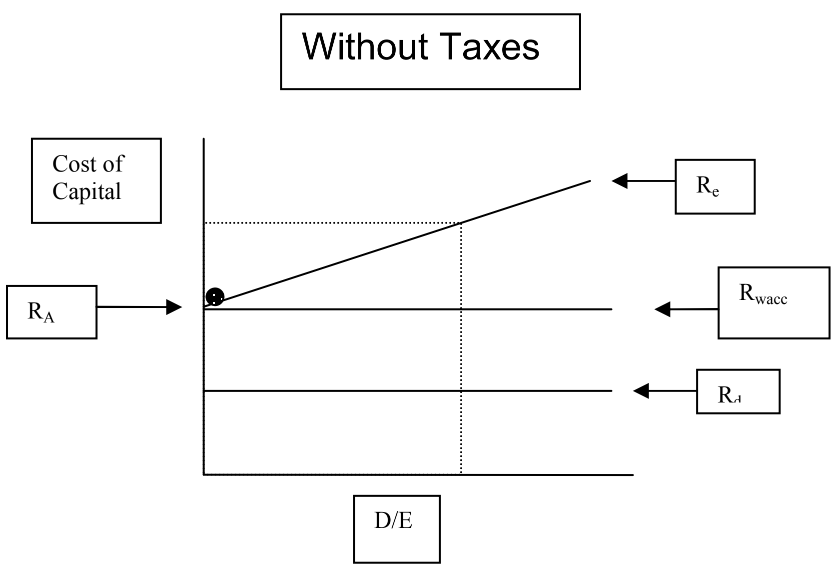
The total cost of capital for the firm then is the weighted cost of debt and equity. This is call R_{wacc}.

Formula
$R_{wacc} = r_s * E/(D+E) + r_b * (1-Tc) * D/(D+E)$

This formula gives the appropriate discount rate for a firm *provided that the D/E ratio does not change!*

The Effect of Financial Leverage on the Cost of Debt Equity Capital: Graphical Depiction





R_e: The two graphs above show two situations, one world where there are no taxes and one world where taxes exist. Both graphs have R_e increase as D/E increases. This is because equity holders are in a riskier position as debt increases because they get paid second.

R_A: Notice the special point on the y-intercept for both graphs. At this point there is no debt (the firm is unlevered) and WACC=R_e. This return is also called the return on assets → R_A. This is what the firm's returns are without any additional "bang" from leverage. It is the return of the unlevered firm.

Taxes: In the taxes world R_{wacc} decreases because of the benefit of the tax shield. This drop in R_{wacc} will increase the value of the company (we will show this later on in the section).

No Taxes: In the world without taxes debt does not generate a tax shield. Since debt does not provide an added bonus, R_{wacc} does not fall.

R_d: Note, in both of these graphs we let R_d remain constant. In reality as D/E increases R_d would also increase to compensate for the increased risk debt holders would be exposing themselves to.

Calculating Rd:

The R_d is the yield to maturity of the debt. If you have a 12% coupon bond ($1000 par) and a 20- year life and a similar bond is trading at $1170.3 in the market the R_d is 10%. Often a computer program calculates YTM. Think of the YTM as the IRR of a bond. Below is an approximation:

YTM ≅ [Coupon + Par – Price)/N]/(Par + Price)/2

120+(1000-1170.3)/20/((1000+1170.3)/2 = .1027

Caveat: Betas Problems

Betas have 3 main problems:

1) They vary overtime
2) Proxies may not be adequate
3) Leverage may change

Problem 1: Betas change over time. To compensate for this error, sophisticated statistical measures must be used to ensure the beta has not changed significantly.

Problem 2: Oftentimes betas are calculated using proxies. The beta of a small shoe company, which happens to be too small to afford a proper beta calculation, may use a rough approximate beta by calculating the unlevered industry beta (a weighted average of Beta assets, also known as unlevered betas, in the industry) and then apply is D/E ratio of the small shoe company. The problem with this is that the company may be involved in many different industries that may bias its beta. For example, if they proxy their beta off of Nike, then they would have to take into account Nike also makes clothes and other non-shoe goods.

Problem 3: Firms changes their D/E from time to time, which will change their beta. To rectify this problem, every time the capital structure changes, beta should be recalculated.

Efficient Market Hypothesis/ Corporate Valuation

In this section we value a company using APV and WACC and also introduce the Efficient Market Hypothesis.

Efficient Market Hypothesis

There are three forms of market efficiency:

1) Weak
2) Semi-strong
3) Strong

We have been implicitly assuming efficient perfect capital markets thus far, and will continue to unless otherwise stated. The efficient market theories are a way of relating information and efficiency in the capital markets. Note: An efficient capital market may not necessarily be a perfect capital market. Perfect capital markets mean that there are not transaction costs: such as transportation costs.

Information Sets

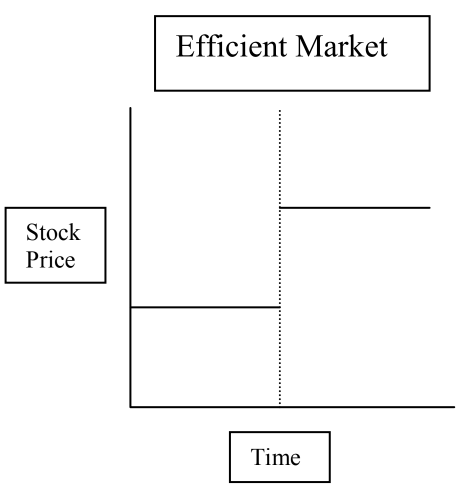
Each of the three forms of efficiency have an information set they are based upon:

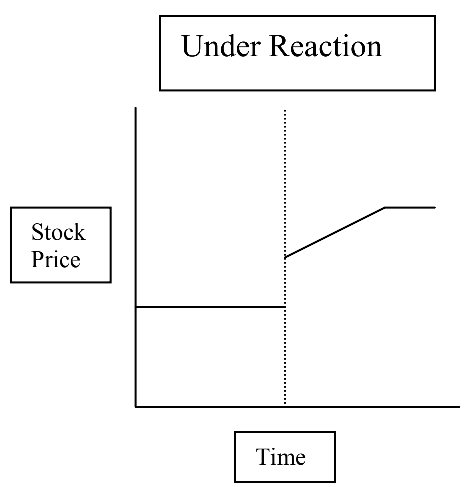
1) Weak: current stock prices reflect past prices
    a. Random Walk
2) Semi-Strong Efficient Market: Prices reflect public information
3) Strong Efficient: Prices reflect all information (public and private)

What efficient market means:

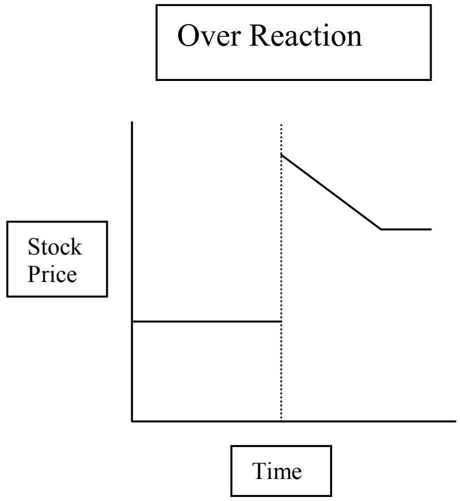
Efficient capital market means that investments in financial securities are 0 NPV projects. What you paid for the stock equals the discounted cash flows you are to receive from the stock.

Also when new information comes out about a stock, the stock immediately adjusts to a reflect the new information fully. There is no time lag between information dispersal and response. There is no under-reaction (say a new favorable earnings report came out the stock price would first from $5 to $10 and later go to $15). Conversely there is no over reaction.





The hashed line represents the time when information is released



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Tests of the EMH (Efficient Market Hypothesis)

Weak: If the weak form exists then you would be able to predict stock prices based on historic prices (this is what technical analysis does). When past stock prices indicate future stock prices, it is said that there is serial correlation in effect. Empirical testing shows that the weak form does not hold.

Semi-strong: Delayed or over reaction to public information implies that the semi-strong form does not hold. If new information comes out and the stock price does not change, this does not necessarily mean that the semi-strong hypothesis was violated. It could mean that the information was immaterial. Data is up for debate about the acceptance/rejection of this form.

Strong: Private information does not play a role in determining stock price. That means insider trading should not be illegal because it will not lead to abnormal returns. Strong form states all information is incorporated in the stock price.

Common Misconceptions:[5]

1) No upward trend in stock prices
2) Investors cannot earn any return
3) Investors should throw darts to select stocks
4) Daily price fluctuations are inconsistent with the EMH
5) There are too few stock holders to achieve inefficiency

Corporate (or Project)[6] Valuation

The simplest way to value a company is to add the market value of debt and equity.

Formula
E+D = V

The market value of equity can be calculated by finding the stock price and multiplying by the number of shares outstanding. We use the market value of equity because as EMH dictates, this will give us the most accurate value of the equity for the firm.

Usually the book value of debt is a good enough proxy for the true value of debt. To get the exact value of debt, take the present value of all bond and principal payments of the debt.

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[5] Adopted from *Corporate Finance* 5th edition by Ross, Westerfield and Jaffe
[6] For fluidity of reading we will just say company and leave out the parenthetical 'for project'. This analysis can be used for projects as well.

Adjusted Present Value (APV) Approach to Valuation

Recall from "**The Effect of Financial Leverage on the Cost of Debt Equity Capital**: Graphical Depiction" section that R_A is the return on an unlevered firm. The difference between a levered and unlevered firm is the present value of the tax shields.

Formula:
$V_L = V_{UL}$ + PVTS + Fin

V_L= Value of a levered firm
V_{UL} = Value of an unlevered firm
PVTS = Present value of the tax shield
Fin= Financial costs which are described under 3 broad groups

a. Cost of issuing new securities: these are payments to bankers for their work (this is subtracted)
b. Cost of financial distress (this is subtracted)
c. Subsidies: This comes from tax free debt issuances (this is added)

1) To perform the APV approach you first calculate UFCF (unlevered free cash flows)[7]:

UFCF = EBIT (1-Tc) + Depreciation – Capital Expenditures (also called increase in PPE) – Increase in Net working capital (NWK).

2) Next NPV the UFCF. This is the PV of the UFCF minus the cost of the project.

3) Add the side effect of the Tax Shield:

Formula:

PVTS= $TcDR_d/(1+R_d)$

If this is a perpetuity then we can simply use $TcDR_d/R_d$ (R_ds cancel out) ➔ TcD.

4) Then add and subtract the appropriate other side effects after present valuing them

WACC Approach to Valuation

1) To use the WACC approach to valuing a company first calculate WACC.

Formula
$R_{wacc} = r_s * E/(D+E) + r_b * (1-Tc) * D/(D+E)$

2) Calculate the UFCF.

3) Take the NPV of the investment. Discount the cash flows by WACC and subtract the initial investment.

Caveats:

1) WACC should only be used if the project we are investing in has the same systematic business risk as the firm.
2) WACC and the project should have the same D/E ratio
3) The D/E ratio should not change over the valuation period

[7] This is gone over in more detail in the cash flows section

When to use APV and WACC?

Use APV when D/E is changing every year. Use WACC when D/E is constant. In most cases you should be able to use both APV and WACC (if D/E is changing you will have to calculate a new WACC for every new ratio, this is tedious but can be done). Both APV and WACC should give you equal values.

The Terminal Value

When valuing a company, you will be given a pro forma (a sheet that predicts the revenues, expenses, etc.) for the next few years, lets just say we have a pro forma sheet for the next 5 years. At the end of the next 5 years assumptions are made about the final value. This value is called the terminal value. Firms have, by definition, an infinite lifespan but it is not possible to meaningfully predict what revenues will be after a certain number of years. When we value a company we are valuing all future free cash flows of the company, but this is an unreasonable task to perform. To partially correct for this problem, assumptions are made about the residual (or terminal) value of the firm. They are listed below. Note the years prior to the terminal value are discounted regularly and not treated as perpetuities.

1) **Growth:** Lets say a firm took a new project. This project will affect cash flows in years described in the pro forma and hence the pro forma takes them into account. After that period, we can assume the company is stable and will grow at a constant rate (usually around inflation). The terminal value is given by a growing perpetuity formula: $FCF/(R_{Wacc} – g)$. 'g' is the growth rate. The growth rate used should be chosen carefully, an unreasonably high growth rate will imply the firm is growing faster than the US economy. This will cause the firm to be larger than the economy –clearly a foolish assertion.
2) **No Growth:** This method uses the perpetuity of above, but assumes no growth in cash flows. This is a more conservative approach. Usually we expect the firm to at least grow with inflation.
3) **Liquidation Value**: This value is what the firm would be worth if it was sold off piece by piece. This is fine for a distressed firm, but for a healthy firm you will grossly underestimate the synergies of operation. A healthy firm should me worth more than what it is composed of. To make a simple analogy, if a baker bakes a cake, you would hope the cake can be sold for more than the cost of the ingredients.
4) **Book Value:** This also yields a conservative value for the reasons in liquidation value.
5) **Multiple Approach:** Sometimes you can use a multiple to calculate the value. You should be careful that multiples are rough approximations. Pay special attention to the approach you are using and the logic behind it.

Example:[8]

Year	1	2	3	4	5	6
EBIT	107	105	102	103	103	108
Tax @ .4	43	42	41	41	41	43
After Tax	64	63	61	62	62	65
Depreciation	165	175	170	145	140	140
CAP Expenditures	50	50	60	130	140	144
NWK	-5	-5	0	4	6	2
FCF	184	193	171	73	56	59

[8] Adopted form Analysis for Financial Management 6th edition by Robert C. Higgins

PV of FCF 1-5 @ 12% wacc = 518

Terminal Value Estimate:
(The value (TV) is at year five. This must be discounted to the present time)

Method *TV*
Perpetual Growth @ 5% = 843

PV of TV = 482.
Value of firm = 1000 (TV + PV of years 1-5)

If the value of debt is 500 equity value is 500. We can divide the equity value by number of shares outstanding to get the price per share.

Options

Deratives: Options are some of the most complex and interesting instruments of modern finance. They are called derivatives because their value is derived from another underlying asset, such as stocks.

Option Jargon

Options: Contracts that let you buy or sell a fixed number of stocks at a fixed price by a certain date

Put Option: Right to sell an underlying asset at a fixed price

Call Option: Right to buy an underlying asset at a fixed price

Striking/Exercising Price: This is the price you are guaranteed to buy or sell the underlying securities at

Exercising the Option: This means you will buy or sell the option; i.e. if it is a call option you will by the option at the fixed price

Expiration Date: Final date to use the option

American Option: Can be exercised any day prior to the expiration date

European Option: Can only be exercised on the expiration date

Long: If you are long a position, then if that item increases in value you gain. (If you are long a stock you are essentially investing in that stock)

Short: As the value of asset decreases you gain money –you are betting against the asset

Dead option: Option that has expired and was not executed –the option has zero value

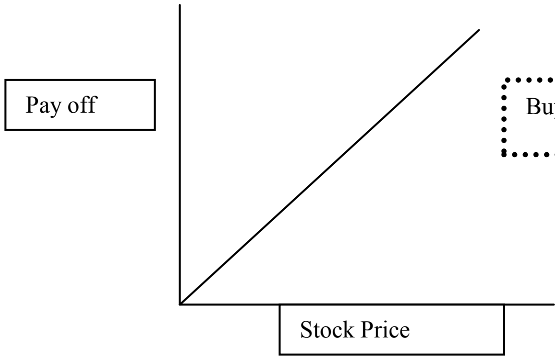
In the money: A call option is in the money if the price of the stock is below the exercise price. A put option is in the money if the price of the stock is below the strike price

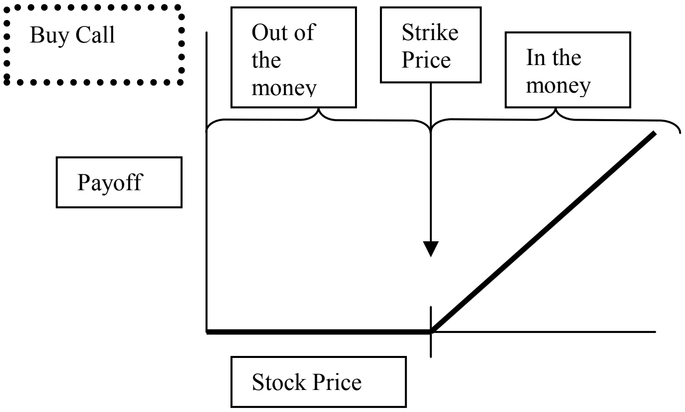
Out of the money: A call option is out of the money if the price of the stock is below the strike price. A put option is out of the money if the price of the stock is above the strike price

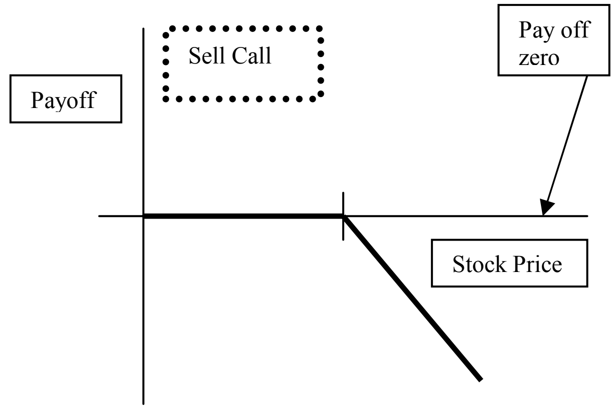
Puts and Calls and Shares

Calls

A call lets the owner buy a stock at a pre-determined price before a pre-determined time. The buyer of the option makes money if the price of the stock exceeds the strike price. The seller of the option makes money if the option dies, because he was paid a fee when he sold the options. Below are the pay off diagrams. A call is in the money when it exceeds the strike. Pay off lines are in bold. Before we look at call options, we will first look at the pay off diagram of buying a regular stock. Use this as a comparison measure for the other pay off diagrams.

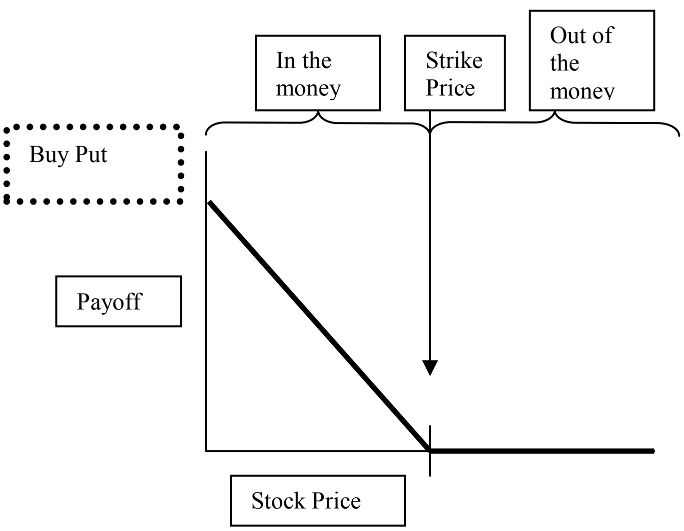


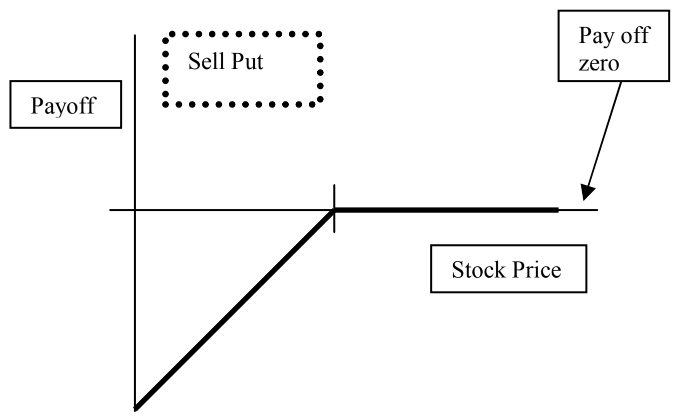




Puts

A put lets the owner sell a stock at a pre-determined price before a pre-determined time. The buyer of the option makes money if the strike price exceeds the stock price. The seller of the option makes money if the option dies, because he was paid a fee when he sold the options. Below are the pay off diagrams. A put is in the money when it is below the strike. Pay off lines are in bold.





Intrinsic and Time Value:

These diagrams tell us several important things. One is that the minimum value of either a call or put option is 0. Also it appears that calls can have infinite pay off diagrams (the price of the stock could theoretically go to infinity) while puts have a limit on their pay off (the y intercept, when a stock is worth zero). Options have two parts to them that determine their value:

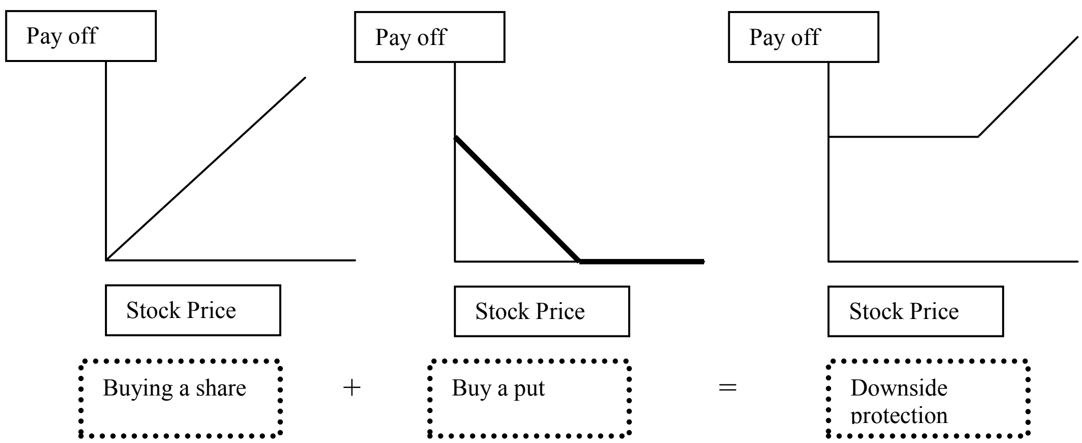
Intrinsic value: This is the value of the option if it was executed today. This value will be zero for out of money options and positive for in the money options.

Time value: This part of an options value comes form the possibility the option will be in the money at some point in the future. It is the holding value of the option.

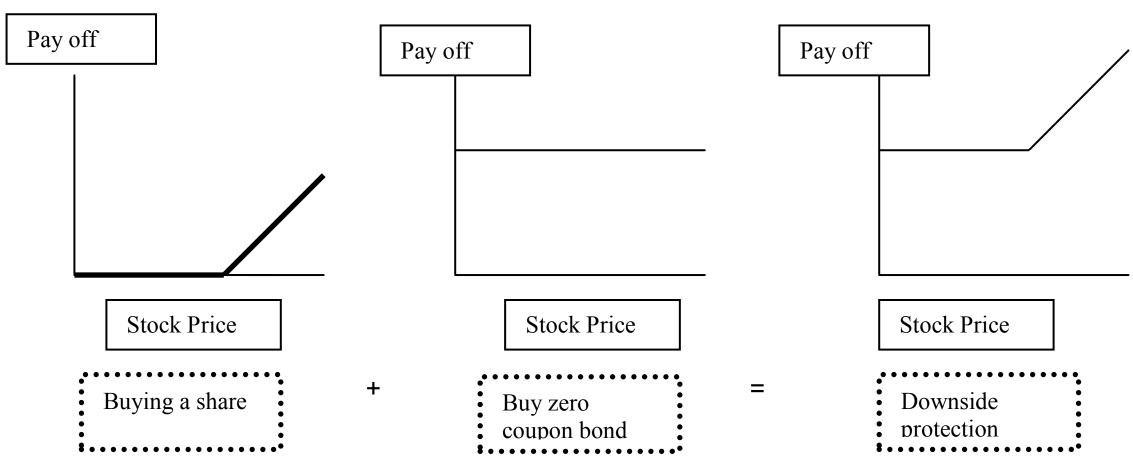
Think of options as an outside bet on the price of a stock. When options are bought and sold no new shares are issued.

Hedging/Combinations

Options are often used as hedges. You can combine puts, calls, risk free assets, and stocks to form interesting pay off schemes, or to hedge different schemes. Below is just one example.



Or you can replicate the payoff by buying a call and a zero coupon bond.



Put Call Parity:

The above exercise shows the relation ship between puts, calls, and the underlying stock. We can sum this relationship as the put-call parity.

Formula:
Value of underlying asset (stock) + Value of put – Value of a call = PV of strike price (discounted by R_f)

Valuing an Option

All of this analysis begs the question, how do we value options? Below is a list of 5 factors and how they affect the price of an option. We are assuming that all of the "Factors" are increasing. That is Time to Expiration has lengthened (all things equal, the expiration date of an option is farther away).

Factors	**Call Option Value**	**Put Option Value**
Stock Price	Increase	Decrease
Strike Price	Decrease	Increase
Risk Free Rate	Increase	Decrease
Volatility of Stock	Increase	Increase
Expiration Time	Increase	Increase/Ambiguous

Note that if all the factors were decreasing the opposite results would ensue. Most of these factors are intuitive, but some are not those are explained below:

Risk Free Rate: Look at the put call parity. Rearrange the variables so that value of the put is the only left hand side variable. By increasing the discount rate we decrease the present value of the strike price, hence decreasing the value of the put. The opposite happens for a call.

Expiration Time: This is an interesting case. It will always increase the value of a call, but may decrease the value of a put. Increasing time to expiration decreases the PV of the strike (as above) thus possibly lowering the value of a put, but increasing the value of a call. Also the longer time increases the chances of favorable outcomes for both puts and calls. This makes the call case unambiguously better, but the put case slightly worse.

Volatility of Stock: With volatility of stock increasing, it would ordinarily mean the pay offs are riskier. But note, the payoff diagram of a stock option is not symmetric. The bottom value of an option is 0 while the maximum value can be anything in terms of a call. Puts also have asymmetric payoffs. This means we cannot use standard deviation as the only proxy for risk. The greater the volatility the more likely the option will rebound in the money. Thus increased volatility increases the value of puts and calls.

Corporate Valuation

The put-call parity (PCP) has other uses as well. For example it can be used to described the situation of a firm. Note that debt holders get paid first, so equity holders have a call on the firm. The strike price is whatever the debt holders are owed.

We rewrite the put – call parity to reflect a corporation:

Value of underlying asset = Value of a call + PV of strike price - Value of put

Value of underlying asset = Value of the firm
Value of the call = Value of equity holder's position
PV of strike price - Value of put = Value of the debt holders position

Note in this interpretation of PCP as volatility increases, the value of the debt holder's position decreases, while the value of the equity holder's position increases.

PV of strike price - Value of put = Value of the debt holders position

The debt holder's position decreases because the value of the put increases dude to increased volatility. Note the value of the put is *subtracted* form the PV of the strike price.

Value of the call = Value of equity holder's position

The increased volatility increases the value of the call, which is the equity holder's position.

<u>What does this mean?</u>

Assume: D = 500
E = 500

You have two investment choices: Project A that pays 500 guaranteed and Project B that pays 0 with a 80% probability and 1000 with a 20% probability.

The expected value of A is 500. The expected value of B is 200.

While, debt holders would prefer A, because that would enable them to get paid regardless of the situation, equity holders will try to chose B. They have a chance of getting 500 with B (though its only 20%) but they have no chance of making any money from A (all the money will go to the debt holders). Since equity holders control management, they may force management to take B. They gain value by stealing it from the debt holders. Project B is riskier than A and has a lower expected value, hence it should not be taken.

This example illustrates the debt holder-equity holder dilemma. To protect themselves, debt holders issue covenants on their debt that prohibit such decisions from being made.

Black-Scholes (B-S) Option Pricing Model

The B-S model is a way to price options. It is probably the most famous and most used method for pricing options. The N(d1) and N(d2) terms can be looked up in a normal distribution table. Before displaying the formula we will first list the assumptions B-S makes. B-S gives you the value of a call option, you can find the value of the put by PCP.

1) No restrictions on short selling
2) No transaction costs
3) No taxes
4) European Option
5) No dividend
6) Stock price is continuous
7) Market operates continuously
8) R_f is a known constant
9) Stock price is lognormally distributed

The Model:

$$C = SN(d_1) - Ke^{(-rt)}N(d_2)$$

C = Theoretical call premium

S = Current Stock price

t = time until option expiration

K = option striking price

r = risk - free interest rate

N = Cumulative standard normal distribution

e = exponential term (2.7183)

$$d_1 = \frac{\ln(S/K) + \left(r + s^2/2\right)t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

s = standard deviation of stock returns

$\ln$ = natural logarithm

The first part, SN(d1), calculates the expected benefit from acquiring a stock outright. This is found by multiplying S by N(d1) –the change in the call premium with respect to a change in the underlying stock price. Ke(-rt)N(d2), gives the present value of paying the exercise price on the expiration day. The fair market value of the call option is calculated by taking the difference between these two parts.

Note: There are alternative versions of the Black-Scholes model to adjust for certain assumptions (such as the no dividend assumption), but we have not included these versions. A common way of adjusting the model for this situation is to subtract the discounted value of a future dividend from the stock price.